Relativistic Electron-Positron Gamma Ray Laser

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The prerequisite for an efficient electron-positron gamma ray laser, which is the rapid formation of a dense electron-positron plasma in a time shorter than the time for pair annihilation, is ideally fulfilled in a relativistic electron-positron superpinch. Because the cross section for annihilation decreases quadratically with the center of mass energy, the time requirements otherwise imposed, are greatly relaxed. A relativistic electron-positron pinch can collapse under a complete population inversion into a very dense state possessing the form of a long filament, just as it is required for a gamma ray laser. The gamma ray energies are the total center of mass energies, which can be much larger than the electron-positron rest mass energies.

To establish a non-relativistic electron-positron plasma with the number density n, the formation of the plasma must take place in a time shorter than the time for pair annihilation

$$t_a \cong 2/n \sigma c, \tag{1}$$

where $\sigma \cong \pi r_0^2$ ($r_0 = e^2/mc^2$ classical electron radius, c velocity of light) is the cross section for pair annihilation. Taking the minimum value of $n \cong 4 \times 10^{20}$ cm^{-3} computed by Bertolotti and Sibilia [1] to be needed for a nonrelativistic electron-positron laser, it follows that the time for formation must be less than $t_a \sim 10^{-6}$ s. There appears, therefore, to be little hope to reach a usable population inversion in a nonrelativistic electron-positron plasma. However, it was pointed out by the author [2] several years ago that a large population inversion can be achieved in a relativistic electron-positron plasma because there the cross section for pair annihilation is greatly reduced. Moreover, in such a relativistic electron-positron plasma one can reach very high densities without destroying the population inversion in the time it is established. Furthermore, through the linear geometry of the pinch configuration the population inversion is reached in a long dense filament, just as it is required for a gamma ray laser.

The idea emerged as a byproduct out of a concept to reach ultrahigh densities in a collapsing relativistic electron-positron superpinch. The prop-

erties of such relativistic superpinches were analyzed by Meierovich [3] who also showed that they are expected to be quite stable [4]. The superpinch is formed through the coalescence of two counterstreaming relativistic electron and positron beams of equal energy and intensity (see Fig. 1), where the electrons and positrons equally contribute to the total electric current I. If such a relativistic pinch configuration has been established, it will shrink in its diameter if the current is larger than some minimum current $I_{\min}$, the reason being that above this minimum current the electron-positron plasma will lose more heat through the emission of synchroton radiation than it is gaining by collisions between the electrons and positrons. The frequency of the synchrotron radiation always matches the plasma frequency, and the plasma remains therefore transparent to the emitted synchroton radiation, regardless of the density the plasma has reached. As a result, the pinch configuration collapses into a final state of very high density, ultimately determined by the laws of quantum mechanics. However, for the pinch to have an equilibrium, the total current must be less than a maximum current $I_{\max}$. It turns out that below this current the kinetic particle energy is always larger than the magnetic field energy inside the pinch. For the minimum and maximum currents one finds

$$I_{\min}/I_A = (9/4 \, \gamma^2) \ln \Lambda,$$
$$I_{\max}/I_A = \gamma, \tag{2}$$

where $I_A = mc^3/e \cong 17000$ A, $\Lambda = r/r_0$, r is the pinch radius, and $\gamma = (1 - v^2/c^2)^{-1/2}$, v is the electron and positron velocity. One typically has $(9/4) \ln \Lambda \sim 100$.
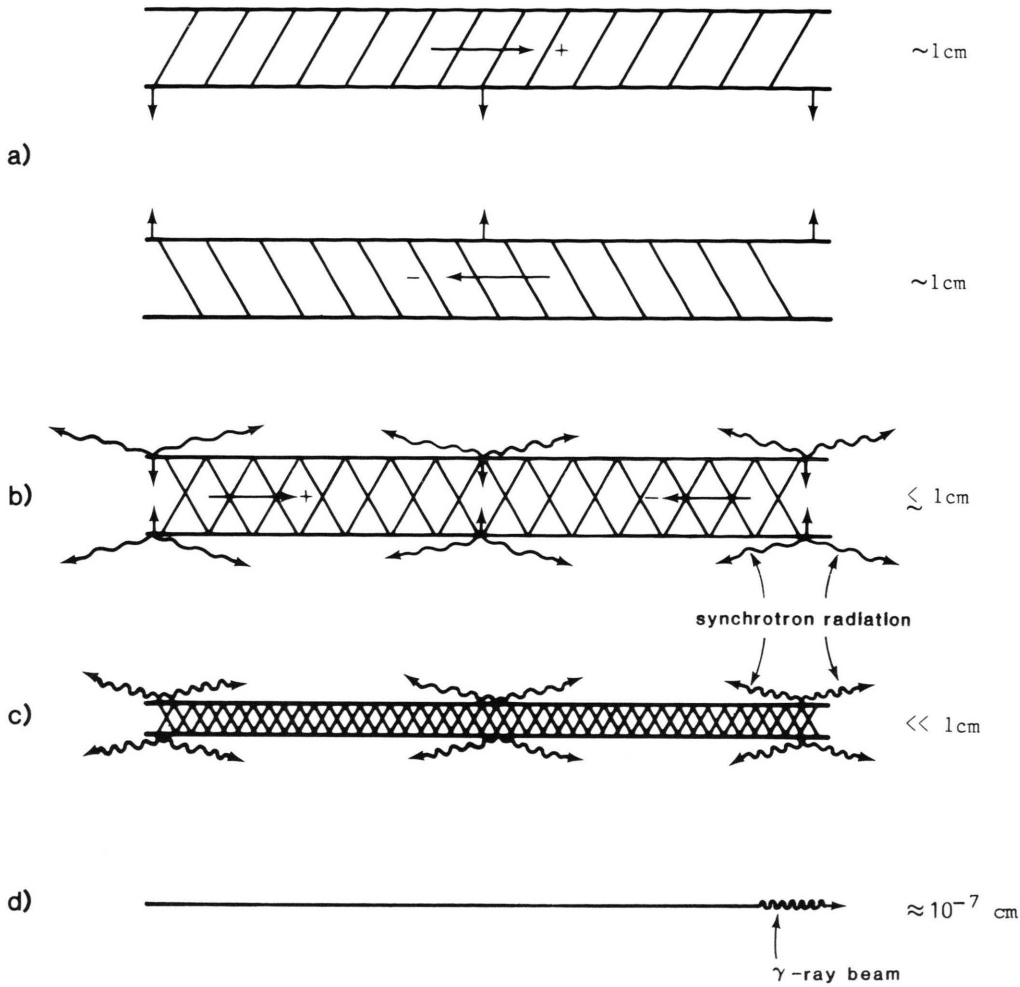
Fig. 1. Coalescence and radiative collapse of two counterstreaming relativistic electron and positron beams to form an electron-positron gamma ray laser with a high population inversion density.

One furthermore computes for the average radial velocity $v_\perp$:

$$v_\perp^2 / c^2 \cong \gamma I / I_A, \quad I \ll \gamma I_A. \tag{3}$$

For $I \ll \gamma I_A$ one always has $v_\perp / c \ll 1$. In reaching the limit $I = \gamma I_A$, (3) has to be replaced by

$$v_\perp^2 / c^2 = (\gamma I / I_A) \sqrt{1 - v_\perp^2 / c^2}, \quad \gamma \gg 1, \tag{3a}$$

where the factor $\sqrt{1 - v_\perp^2 / c^2} = \cos \alpha$ takes into account that the particles move in trajectories declined by the angle α against the pinch axis. For $I = \gamma I_A$, (3a) yields the golden cut value $v_\perp^2 / c^2 = (\sqrt{5} - 1)/2 = 0.62$, implying $v_\perp / c = 0.79$, and $\alpha = 52°$. From this value follows $v_\parallel = c \cdot \cos \alpha = 0.62\,c$. This veloc-

ity is sufficiently small to permit a photon avalanche propagating with the velocity of light through the pinch channel. Due to the large radial velocity the electric current is reduced also by the factor $\cos \alpha = 0.62$.

During its collapse to a smaller radius the pinch is the source of highly coherent synchrotron radiation, emitted under the angle $\sim 1/\gamma$ and with an energy given by

$$\hbar \omega_s = \hbar \sqrt{2/\gamma} \, (c/r) \sqrt{I/I_A}. \tag{4}$$

For $I \cong \gamma I_A$ in particular one has $\hbar \omega_s \cong \hbar c/r$. Depending on the smallness of r reached during the pinch collapse, the wavelength of the emitted syn-

chrotron radiation can become very short. Near the quantum limit, which for example can be reached at $r \sim 10^{-12}$ cm, one finds that $\hbar\omega_s \sim 20$ MeV.

The e-fold time needed for the pinch of initial plasma radius r to collapse was computed to be

$$t_0 = (9/8)\,(r^2/r_0\,c\,\gamma^2)\,(I_A/I)\,. \qquad (5)$$

This time has to be compared with the time for pair annihilation. In case of relativistic electrons and positrons colliding head on, the annihilation cross section is

$$\sigma \cong (\pi\,r_0^2/\gamma^2)\,\ln(2\gamma)\,. \qquad (6)$$

Compared to the nonrelativistic situation the cross section is therefore roughly reduced by the factor $1/\gamma^2$. This large reduction in the cross section combined with the rapid pinch collapse is the reason why under these circumstances a large population inversion at a high density can be achieved.

With the cross section given by (6) we find for the annihilation time

$$t_a = (2/n\,\sigma\,c) = [(2\,r\,\gamma)^2/(c\,r_0\,\ln(2\,\gamma))]\,(I_A/I)\,, \qquad (7)$$

such that

$$t_0/t_a = (9/32)\,(1/\gamma^4)\,\ln(2\,\gamma)\,, \qquad (8)$$

independent of r and I. This ensures that even for modestly large γ-values always $t_0 \ll t_a$.

To employ the configuration this way established as a gamma ray laser, it is required that the axial motion of the electrons and positrons is greatly reduced. Only then is a laser action by a photon avalanche along the pinch channel possible. According to (3) this requires to put $I = \gamma I_A$. The gain which can be reached in a plasma column of length l is then computed to be

$$G = n\,\sigma\,l/2 = [(r_0\,l\,\ln(2\,\gamma))/4\,r^2\,\gamma]\,. \qquad (9)$$

The gamma ray photon energy released per unit length of the pinch channel is

$$E_p = 2\,n\,\pi\,r^2\,(\gamma - 1)\,m\,c^2 \cong m\,c^2\,\gamma^2/r_0\,, \quad (\gamma \gg 1)\,. \qquad (10)$$

If for example $l = 100$ cm and $\gamma = 20$ (10 MeV electrons and positrons), we compute $G = 100$ for a pinch radius $r \sim 10^{-7}$ cm. In this example $I \cong 340\,000$ A, and there are $\sim 7 \times 10^{15}$ electron-positron pairs, each releasing a photon energy of ~ 20 MeV. By comparison, the energy of the syn-

chrotron radiation emitted is only ~ 200 eV, which is in the soft X-ray domain. The 7×10^{15} pairs annihilated release a total energy of ~ 20 kJ.

It may be possible, but is not yet explored, that the very large self-magnetic beam field, which for $I = 3.4 \times 10^5$ A and $r \cong 10^{-7}$ cm is $H_\varphi \sim 10^{12}$ Gauss, can carry a recoil momentum. In this case all the energy released during annihilation could go into one 20 MeV gamma quant, rather than two 10 MeV quants moving in opposite directions.

During the collapse of the pinch to a smaller radius the magnetic energy outside the pinch column increases. This energy must be taken from the energy stored in the pinch channel. The energy in the pinch channel is the sum of the kinetic particle energy and magnetic energy. The kinetic particle energy per unit pinch length is

$$E_{kin} = (\gamma\,m\,c^2/r_0)\,(I/I_A)\,. \qquad (11)$$

The magnetic energy inside the pinch channel is

$$E_H^i = (1/2)\,(m\,c^2/r_0)\,(I/I_A)^2\,, \qquad (12)$$

and outside the channel

$$E_H^e = (m\,c^2/r_0)\,(I/I_A)^2\,\ln(R/r)\,, \qquad (13)$$

where R is the radius of the return current conductor. To compensate for the rise in the magnetic energy outside the channel we make at the beginning of the collapse $\gamma = \gamma_0$ larger than its final value $\gamma = I/I_A$. With $\gamma_0 \gg \gamma$ one has $v_\perp^2/c^2 = I/\gamma_0 I_A = \gamma/\gamma_0 \ll 1$. Therefore, at the beginning of the collapse I remains unchanged since there $\cos\alpha \cong 1$. The kinetic energy at the beginning of the collapse therefore is

$$E_{kin}^{(0)} = \gamma_0\,(m\,c^2/r_0)\,(I/I_A) = (m\,c^2/r_0)\,\gamma\,\gamma_0\,. \qquad (14)$$

Under the choice whereby the initial pinch radius $r_i \cong R$, the magnetic energy at the beginning of the collapse is

$$E_H^{(0)} = (1/2)\,(m\,c^2/r_0)\,(I/I_A)^2$$
$$= (1/2)\,(m\,c^2/r_0)\,\gamma^2\,. \qquad (15)$$

At the end of the collapse the kinetic energy is

$$E_{kin} = (m\,c^2/r_0)\,\gamma^2\,. \qquad (16)$$

Unlike in the expression for the kinetic energy where the current I is simply a measure of the number of particles, leaving the factor I/I_A unchanged during the pinch collapse, $I = \gamma I_A$, must

there be multiplied by the factor $\cos \alpha = 0.62$. With this adjustment the magnetic energies, inside and outside the pinch, and at the end of the collapse are

$$E_H^i = (1/2)\,(m\,c^2/r_0)\,(0.62)^2\,\gamma^2 = 0.193\,(m\,c^2/r_0)\,\gamma^2 \quad (17)$$

and

$$\begin{aligned}E_H^e &= (m\,c^2/r_0)\,(0.62)^2\,\gamma^2 \ln (r_i/r)\\ &= 0.384\,\gamma^2 \ln (r_i/r)\,.\end{aligned} \quad (18)$$

From the energy conservation equation

$$E_{kin}^{(0)} + E_H^{(0)} = E_{kin} + E_H \quad (19)$$

follows that

$$\gamma_0/\gamma = 0.693 + 0.384 \ln (r_i/r)\,. \quad (20)$$

For the example $r_i \sim 1$ cm, $r \sim 10^{-7}$ cm follows $\gamma_0/\gamma \sim 7$. Therefore, if $\gamma \cong 20$ then $\gamma_0 \cong 140$, which corresponds to 70 MeV electrons and positrons. The maximum current $I = \gamma I_A \cong 340\,000$ A is well above the minimum current given by (2), which for the example given is $I_{min} \cong I_A/4 \sim 4000$ A. In the moment of maximum pinch contraction the curent is reduced by the factor $\cos \alpha$ to $\cong 210\,000$ A. The electron-positron plasma would therefore have to be established by the coalescence of two 170 000 A 70 MeV electron-positron beams.

[1] M. Bertolotti and C. Sibilia, Appl. Phys. **19**, 127 (1979).
[2] F. Winterberg, Phys. Rev. A **19**, 1356 (1979).
[3] B. E. Meierovich, Phys. Rev. A **26**, 1128 (1982).
[4] B. E. Meierovich, J. Plasma Physics **29**, 361 (1983).